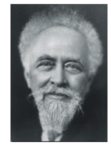
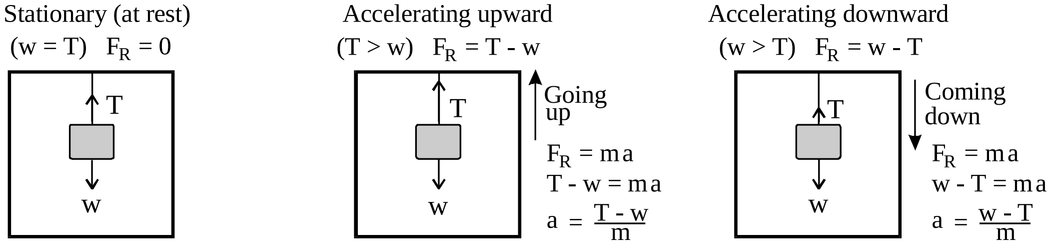


PERRIN

Jean Perrin was awarded the Nobel prize for physics in 1926 for his work on sedimentation equilibrium.

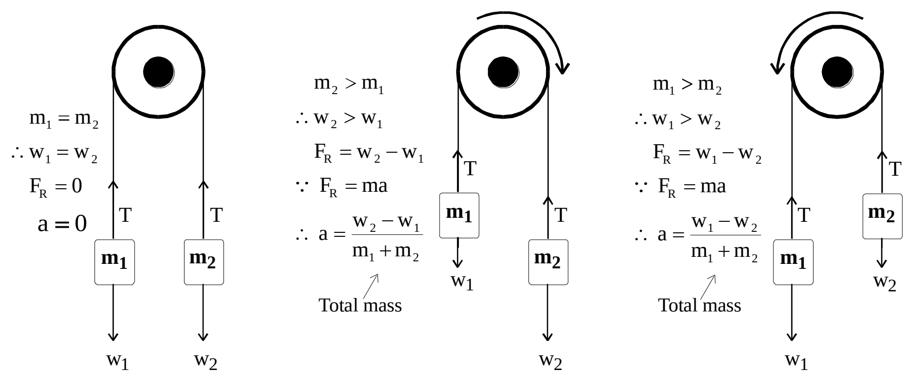
Suspended Accelerating-Objects

A resultant force causes a system to accelerate. The direction of the acceleration is in the direction of the resultant force. As illustrated below, when suspended objects accelerate, they do so in the direction of the resultant force.



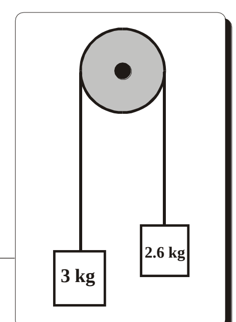
Frictionless Pulley

When two masses are attached at opposite ends of a "light" cord that passes over a frictionless pulley, the two masses act as a single mass whose acceleration is as follows:



➲ **Note**: **Use 10 m/s² for the acceleration due to gravity.**

1. A ***light*** (i.e. massless) cord passes over a pulley that turns on a frictionless axis as illustrated in the diagram on the right. Two masses, 3 kg and 2.6 kg, are attached at opposite ends of the cord. Determine the ***magnitude*** of the acceleration of the masses. [0.7 m/s²]



Note that the cord unites the objects into a single mass of 5.6 kg.

$$\mathbf{F_R = w_1 - w_2 = m_1 g - m_2 g = (3\,kg)(10\,m/s^2) - (2.6\,kg)(10\,m/s^2) = 4\,N}$$

$$\mathbf{\because\ F_R = ma}$$

$$\mathbf{\therefore\ a = \frac{F_R}{m} = \frac{4\,N}{5.6\,kg} = 0.7\,m/s^2}$$

2. A stone is mechanically shot straight up in the air with a velocity of 30 m/s.

$v_i = 30 \text{ m/s}$

$v_f = 0$

a) How *high* does the stone rise? [45 m]

$$v_a = \frac{v_f + v_i}{2} = \frac{0 + 30 \text{ m/s}}{2} = 15 \text{ m/s}$$

$$s = v_a t = (15 \text{ m/s})(3 \text{ s}) = 45 \text{ m}$$

b) At what time will the stone be 40 m from the ground on its way *up*? [2 s]

$$s = v_i t + \frac{1}{2} a t^2$$

$$\text{or} \quad 40 = 30\,t - 5t^2$$

$$5t^2 - 6t + 8 = 0$$

$$(t - 2)(t - 4) = 0$$

$$\therefore \; t = 2 \text{ s} \quad \text{and} \quad t = 4 \text{ s}$$

Answer : $t = 2 \text{ s}$ **(on its way up)**

c) At what time will the stone be 40 m from the ground on its way *down*? [4 s]

$$\therefore \; t = 2 \text{ s} \quad \text{and} \quad t = 4 \text{ s}$$

Answer : $t = 4 \text{ s}$ **(on its way down)**

$$s = v_i t + \frac{1}{2} a t^2$$

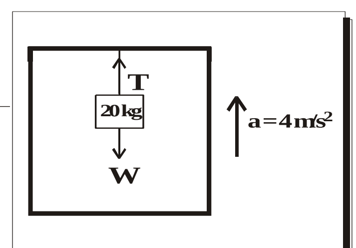
$$\text{or} \quad 40 = 30\,t - 5t^2$$

$$5t^2 - 6t + 8 = 0$$

$$(t - 2)(t - 4) = 0$$

3. A mass of 20 kg hangs by a rope from a ceiling inside an elevator.

a) What will be the tension in the rope if the elevator is accelerating ***upward*** at 4 m/s^2? [280 N]



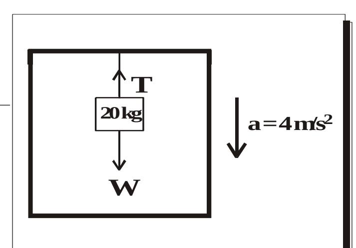
Since the elevator is going up : $F_R = T - w$

$\because \; F_R = ma$

$\therefore \; T - w = ma \quad \text{or} \quad T - mg = ma$

$$T = ma + mg = (20 \text{ kg})(4 \text{ m/s}^2) + (20 \text{ kg})(10 \text{ m/s}^2) = 280 \text{ N}$$

b) What will be the tension in the rope if the elevator is accelerating ***downward*** at 4 m/s^2? [120 N]



Since the elevator is going down : $F_R = w - T$

$\because \; F_R = ma$

$\therefore \; w - T = ma \quad \text{or} \quad mg - T = ma$

$$T = mg - ma = (20 \text{ kg})(10 \text{ m/s}^2) - (20 \text{ kg})(4 \text{ m/s}^2) = 120 \text{ N}$$

4. A stone is thrown vertically upward with a velocity of 15 m/s from the bottom of a cliff that is 15 m high. At the same time, another stone is dropped from the top of the cliff. At what height from the ground will the two stones meet? [10 m]

$$s_{Down} + s_{up} = 15\ m$$

$$\text{But } s = v_i t + \frac{1}{2}at^2$$

$$\therefore\ v_{Down} t + \frac{1}{2}at^2 + v_{Up} t + \frac{1}{2}at^2 = 15\ m$$

$$\text{or } 0 + 5t^2 + 15t - 5t^2 = 15$$

$$\therefore\ 15t = 15$$

$$t = 1\ s$$

$$\because\ s = v_i t + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}(10)(1)^2 = 5\ m$$

Thus, the ball falls 5 m after 1 s.

Answer : 15 m − 5 m = 10 m

5. A boy ***drops*** a stone vertically downward from a bridge 125 m high. One second later,

s]

Note that while the distance for both stones is the same (125 m), the times are not the same. If t represents the time of fall for the first stone, then the time for the second stone is (t − 1) since it was thrown one second later.

Time of first stone

$$s = v_i t + \frac{1}{2}at^2$$

$$125\ m = 0 + \frac{1}{2}(10\ m/s^2)(t^2)$$

$$125\ s^2 = 5t^2$$

$$\text{or } t^2 = 25\ s^2$$

$$\therefore\ t = 5\ s$$

Time of second stone

$$t = 5\ s - 1\ s = 4\ s$$

Initial velocity of second stone

$$s = v_i t + \frac{1}{2}at^2$$

$$125\ m = v_i(4\ s) + \frac{1}{2}(10\ m/s^2)(4\ s)^2$$

$$125\ m = v_i(4\ s) + (5\ m/s^2)(16\ s^2)$$

$$4\ v_i = 45\ m/s$$

$$\therefore\ v_i = 11.25\ m/s \text{ or } 11.3\ m/s$$

6. A rock is thrown vertically down with a velocity of 25 m/s. With what velocity will it strike the ground 100 m below? [51 m/s]

$$\because \quad 2as = v_f^2 - v_i^2$$
$$\therefore \quad v_f^2 = 2as + v_i^2$$
$$\text{or} \quad v_f^2 = 2(10\ m/s^2)(100\ m) + (25\ m/s)^2$$
$$\text{or} \quad v_f^2 = 2625\ m^2/s^2$$
$$\therefore \quad v_f = 51.2\ m/s = 51\ m/s$$

7. A pebble is dropped from the roof of a high building. If it takes 1.5 s to travel the ***last*** 100 m, how high is the building? [275.2 m]

Calculation of initial velocity for last 100 m.

$$s = v_i t + \frac{1}{2}at^2 \quad \text{or} \quad 100\ m = v_i(1.5\ s) + \frac{1}{2}(10\ m/s^2)(1.5\ s)^2$$
$$\text{or} \quad 100\ m = (1.5\ s)v_i + (5\ m/s^2)(1.5\ s)^2$$
$$\text{or} \quad v_i(1.5\ s) = 11.25\ m - 100\ m$$
$$\therefore \quad v_i = \frac{88.75\ m}{1.5\ s} = 59.16\ m/s = 59.2\ m/s$$

Calculation of height of top part of building

$$\because \quad 2as = v_f^2 - v_i^2$$
$$\therefore \quad s = \frac{v_f^2 - v_i^2}{2a} = \frac{(59.2\ m/s)^2 - 0}{2(10\ m/s^2} = \frac{3504.64\ m^2/s^2}{20\ m/s^2} = 175.2\ m$$

Thus, the height of the building is: $100\ m + 175.2\ m = 275.2\ m$

8. With what initial upward velocity must a ball be thrown in order to rise 20 m? [20 m/s]

$$\because \quad 2as = v_f^2 - v_i^2$$
$$v_i^2 = v_f^2 - 2as$$
$$v_i^2 = 0 - 2(10\ m/s^2)(20\ m)$$
$$v_i^2 = 400\ m^2/s^2$$
$$\therefore \quad v_i = 20\ m/s$$

9. A package is attached to a helicopter ready to be released (dropped). If the helicopter is ***ascending*** at 5 m/s and releases the package from an altitude of 550 m, how long does it take the package to strike the ground? [11 s]

Since the helicopter is ascending, the acceleration of the package is negative when released.

$$\because \quad s = v_i t + \frac{1}{2}at^2$$

$$550 = (-5)t + \frac{1}{2}(10)t^2$$

or $550 = -5t + 5t^2$

or $t^2 - t - 110 = 0$

$\therefore \quad A = 1 \quad B = -1 \quad C = -110$

Using the quadratic equation

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-110)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 + 440}}{2} = \frac{1 \pm \sqrt{441}}{2} = \frac{1 \pm 21}{2}$$

$$= \frac{1 + 21}{2} = \frac{22}{2} = 11\text{ s}$$ (discard negative value)

10. Starting from rest, a block of wood takes 5 s to slide down an inclined plane 120 cm long.

a) What was its acceleration? [0.1 m/s^2]

$$\because \quad s = v_i t + \frac{1}{2}at^2$$

or $1.2\text{ m} = 0 + \frac{1}{2}(a)(5\text{ s})^2$

$$\therefore \quad a = \frac{2(1.2\text{ m})}{25\text{ m}^2/\text{s}^2} = 0.096\text{ m/s}^2 = 0.1\text{ m/s}^2$$

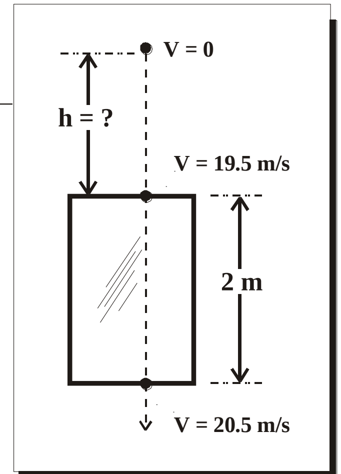
b) With what speed did it reach the bottom of the incline? [0.5 m/s]

$\because \quad 2as = v_f^2 - v_i^2$

or $v_f^2 = v_i^2 + 2as = 0 + 2(0.1\text{ m/s}^2)(1.2\text{ m}) = 0.24\text{ m}^2/\text{s}^2$

$\therefore \quad v_f = 0.489\text{ m/s} = 0.5\text{ m/s}$

11. A ball falls from rest and is seen to pass a window 2 m high in 0.1 s. Determine the height above the window that the stone fell? [19 m]



Initial velocity of ball upon passing top of window:

$$\because \quad s = v_i t + \frac{1}{2}at^2$$

$$\therefore \quad v_i = \frac{s}{t} - \frac{1}{2}\frac{at^2}{t} = \frac{s}{t} - \frac{at}{2} = \frac{2\text{ m}}{0.1\text{ s}} - \frac{(10\text{ m/s}^2)(0.1\text{ s})}{2} = 19.5\text{ m/s}$$

Final velocity of ball upon passing bottom of window:

$$\because \quad a = \frac{\Delta v}{t} \qquad \therefore \quad \Delta v = at = (10\text{ m/s}^2)(0.1\text{ s}) = 1\text{ m/s}$$

But $\Delta v = v_f - v_i$

$$\therefore \quad v_f = \Delta v + v_i = 1\text{ m/s} + 19.5\text{ m/s} = 20.5\text{ m/s}$$

Total distance ball falls:

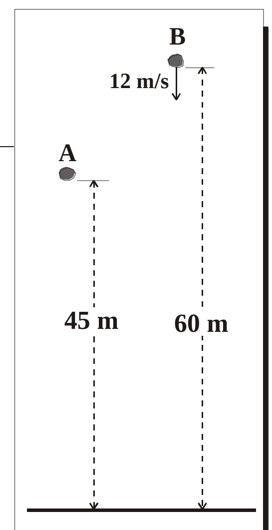
$$\because \quad 2as = v_f^2 - v_i^2$$

$$\therefore \quad s = \frac{v_f^2 - v_i^2}{2a} = \frac{(20.5\text{ m/s})^2 - 0}{2(10\text{ m/s}^2)} = \frac{420.25\text{ m}^2/\text{s}^2}{20\text{ m/s}^2} = 21.0\text{ m}$$

$\because$ Distance above window = Total distance − Height of window

$\therefore$ Distance above window = 21.0 m − 2.0 m = 19 m

12. Two stones, A and B, are respectively 45 m and 60 m above the ground. If stone A is dropped at the same time as stone B is thrown vertically down with a velocity of 12 m/s, which stone hits the ground first? [B]



[illegible]

Time stone – A strikes ground

$$\because \quad s = v_i t + \frac{1}{2}at^2$$

$$45\text{ m} = 0 + \frac{(10\text{ m/s}^2)t^2}{2}$$

$$\text{or} \quad t^2 = \frac{2(45\text{ m})}{10\text{ m/s}^2} = 9\text{ s}^2$$

$$\therefore \quad t = 3\text{ s}$$

Time stone – B strikes ground

$$\because \quad s = v_i t + \frac{1}{2}at^2$$

$$\text{or} \quad 60\text{ m} = (12\text{ m/s})t + (5\text{ m/s}^2)t^2$$

$$\text{or} \quad 5t^2 + 12t - 60 = 0$$

$$\therefore \quad A = 5 \quad B = 12 \quad C = -60$$

Using the quadratic equation

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{-12 \pm \sqrt{(12)^2 - 4(5)(-60)}}{2(5)}$$

$$= \frac{-12 \pm \sqrt{144 + 1200}}{10} = \frac{-12 \pm \sqrt{1344}}{10}$$

$$= \frac{-12 + 36.6}{10} = \frac{24.6}{10} = 2.46\text{ s} = 2.5\text{ s}$$

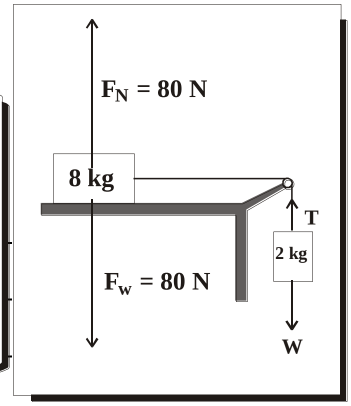
(discard negative value)

13. Two masses, 8 kg and 2 kg, are connected by a light rope that passes

a) Th

$$\because \quad F_R = ma$$

$$\therefore \quad a = \frac{F_R}{m} = \frac{20\ N}{10\ kg} = 2\ m/s^2$$



b) The tension in the cord. [16 N]

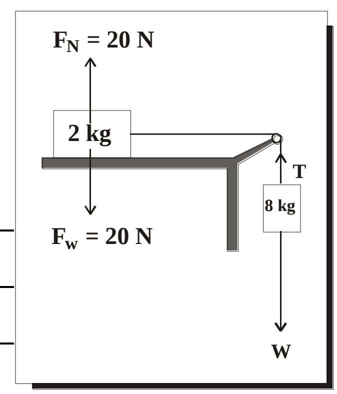
$$F_R = ma$$

$$F_R = 20\ N - T$$

$$\therefore \quad ma = 20\ N - T$$

$$T = 20\ N - ma = 20\ N - (2kg)(2\ m/s^2) = 16\ N$$

14. Two masses, 2 kg and 8 kg, are connected by a light rope that passes over a frictionless pulley (see diagram). Find:



a) The magnitude of the acceleration of the masses. [8 m/s^2]

$$\because \quad F_R = ma$$

$$\therefore \quad a = \frac{F_R}{m} = \frac{80\ N}{10\ kg} = 8\ m/s^2$$

b) The tension in the cord. [16 N]

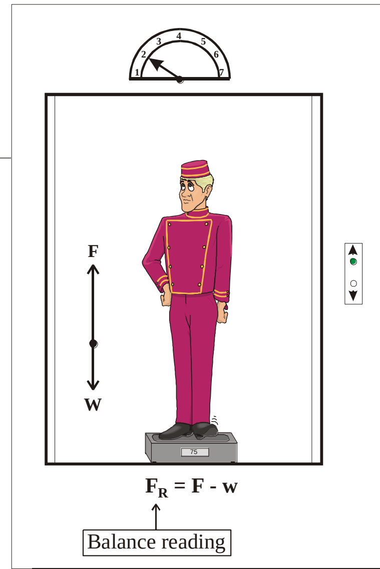
$$F_R = ma$$

$$F_R = 80\ N - T$$

$$\therefore \quad ma = 80\ N - T$$

$$T = 80\ N - ma = 80\ N - (8\ kg)(8\ m/s^2) = 16\ N$$

15. A 60 kg person stands on a spring scale in an elevator. If the elevator is accelerating upwards at 2 m/s^2, what is the reading on the balance? [720 N]



$$F_R = ma$$

But $F_R = F - w$

$\therefore$ $F - w = ma$

$$F - 600\text{ N} = (60\text{ kg})(2\text{ m/s}^2)$$

$$F = 600\text{ N} + 120\text{ N}$$

$\therefore$ $F = 720\text{ N}$

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